Ground Mount Pier Design Optimization

February 13, 2022

Goal: Specify concrete pier depth and diameter for Northern Cheyenne ARPA At-Risk Elder 10-kW solar photovoltaic 15-array Project given local wind and snow conditions.

Parameters and parameter values included:

array elevation angle:	α = 30°, 45°, 60°
array front-edge height:	h = 24”, 48”, 72”, 96”, 120”
pier depth:	Z_P = 3’, 4’, 5’, 6’
number of posts/piers:	n = 10, (12, 14, 16 pending)
pier diameter:	D_P = 1’ (1.5’, 2’ pending)
wind speed:	U = 90, 100, 110, 120 mph

Address the pros and cons of selecting the given values of each parameter.

1. array elevation angle, α = 30°, 45°, 60°
    a. α = 30° pros:
        i. shorter overall array minimizes structural steel costs
        ii. lower profile minimizes wind loads
    b. α = 30° cons:
        i. shallower angle does not promote snow shedding
        ii. shallower angle does not capture winter sun
    c. α = 45° pros:
        i. provides balance between summer production and winter production
        ii. “proven” design
    d. α = 45° cons:
        i. does not maximize winter production
        ii. may not provide adequate snow shedding (Figure 1)
    e. α = 60° pros:
        i. provides greater winter solar production
        ii. provides greater snow shedding
    f. α = 60° cons:
        i. greater material cost due to greater array height
        ii. reduced summer production
        iii. increased risk of wind damage

Figure 1. *Muddy Hall PV array ($\alpha = 45^{o}$, h = 48”) Feb 2, 2022 with significant snow cover.*[1]

2. array front-edge height, h = 24”, 48”, 72”, 96”, 120”
    a. h = 24” pros
        i. provides lowest cost of structural steel
        ii. minimizes wind-load-induced stress
        iii. easiest to build
    b. h = 24” cons
        i. production losses from terrain shading
        ii. possible production losses from snow shading
        iii. production losses from vegetation shading
        iv. production losses from transient shading, i.e. animals or vehicles
        v. may invite unwanted climbing
    c. h = 48” pros
        i. modest cost of structural steel
        ii. modest wind-load-induced stress
        iii. moderately easy to build
    d. h = 48” cons
        i. some production losses from terrain shading
        ii. possible production losses from vegetation shading
        iii. some production losses from transient shading, i.e. animals or vehicles
        iv. may invite unwanted climbing
    e. h = 72” pros
        i. nominal shading losses
        ii. lower climbing risk
    f. h = 72” cons
        i. added structural steel cost
        ii. added wind load
    g. h = 96” pros
        i. nominal shading losses
        ii. lower climbing risk

---

[1] Snow removal service was discussed during Feb 11, 2022 meeting among Bradley Layton, Daniel East, and Sonny BraidedHair.

   h. h = 96” cons
      i. added structural steel cost
      ii. added wind load
   i. h = 120” pros
      i. minimal shading losses
      ii. lowest climbing risk
   j. h = 120” cons
      i. maximum structural steel cost
      ii. maximum wind load
3. pier depth, Z_P = 3’, 4’, 5’, 6’
   a. Z_P = 3’ pros
      i. lowest cost of concrete
      ii. lowest cost of auger time, fuel, and labor
   b. Z_P = 3’ cons
      i. highest risk of pier pull-out
      ii. little room for error in post length
   c. Z_P = 4’ pros
      i. modest cost of concrete
      ii. modest cost of auger time, fuel, and labor
   d. Z_P = 4’ cons
      i. modest risk of pier pull-out
      ii. modest room for error in post length
   e. Z_P = 5’ pros
      i. modest risk of pier pull-out
      ii. room for post length error
   f. Z_P = 5’ cons
      i. concrete cost
      ii. auger time, fuel and labor cost
   g. Z_P = 6’ pros
      i. minimal risk of pier pull-out
      ii. maximum for post length error
   h. Z_P = 6’ cons
      i. maximum concrete cost
      ii. maximum auger time, fuel and labor cost
4. number of posts, n = 10, 12, 14, 16
   a. n = 10 pros
      i. lowest cost of structural steel and concrete
      ii. lowest cost of auger time, fuel, and labor
   b. n = 10 cons
      i. highest risk of structural steel deformation/yield
      ii. highest risk of rail deformation/yield
   c. n = 12 pros
      i. modest cost of structural steel and concrete
      ii. modest cost of auger time, fuel, and labor

d. n = 12 cons
   i. moderate risk of structural steel deformation/yield
   ii. moderate risk of rail deformation/yield
e. n = 14 pros
   i. modest risk of structural steel deformation/yield
   ii. modest risk of rail deformation/yield
f. n = 14 cons
   i. cost of structural steel and concrete
   ii. cost of auger time, fuel, and labor
g. n = 16 pros
   i. minimal risk of structural steel deformation/yield
   ii. minimal risk of rail deformation/yield
h. n = 16 cons
   i. maximal cost of structural steel and concrete
   ii. maximal cost of auger time, fuel, and labor

To perform the optimization, I adopted a code previously written in MATLAB for Montana Solar,[2] and cast the code into MS Excel. To estimate pull-out force, I wrote a system of three equations for static loading and obtained the expected pull-out force for the four [4] wind speeds modeled (Table 1).

Table 1. *pull-out forces in lbs for angle α, windspeed U, and array-front height, h.*

vertical reaction force on south piles as function of front height		90 mph			100 mph			110 mph			120 mph		
		30	45	60	30	45	60	30	45	60	30	45	60
$R_{sy,tot}$	24	23,734	34,176	44,889	29,301	42,193	55,418	35,455	51,054	67,056	42,194	60,758	79,802
	48	25,357	37,084	51,481	31,305	45,782	63,557	37,880	55,396	76,903	45,080	65,926	91,521
	72	26,981	39,991	58,073	33,310	49,371	71,695	40,304	59,739	86,751	47,966	71,094	103,241
	96	28,604	42,898	64,665	35,314	52,960	79,833	42,729	64,082	96,598	50,852	76,263	114,960
	120	30,227	45,805	71,257	37,318	56,549	87,971	45,154	68,425	106,445	53,738	81,431	126,679

As seen in Table 1, the minimum predicted vertical force on the set of southern piers, $R_{sy,tot}(U = 90 \text{ mph}, \alpha = 30^o, h = 24")$ the predicted pull-out force is just under 24,000 lbs, and for $R_{sy,tot}(U = 120 \text{ mph}, \alpha = 60^o, h = 120")$, the predicted pull-out force is just over 125,000 lbs.

Next, we distribute this load over half of the piers, i.e. $n/2$ = 5, 6, 7, 8 and calculate safety factors, SF based on the model published by (Owino, Zakaria, Shiau 2018)[3] for each configuration (Table 2).

---

[2] Montana Solar: https://www.mtsolar.us

[3] Owino, Zakaria, Shiau, 2018: https://www.researchgate.net/publication/328405959_Pull-out_Resistance_of_Single_Piles_and_Parametric_Study_using_the_Finite_Difference_Method_FDM

Table 2. *Safety factor results for 10-post array (n = 10), post diameter, D_P = 1′, post depth, Z_P = 3′, 4′, 5′, 6′, windspeed U = 90, 100, 110, 120 mph, and array front height h = 24″, 48″, 72″, 96″, 120″. Key: SF < 1.0, 1.0 < SF < 2.0, 2.0 < SF < 3.0, ~~SF > 3.0~~.*

	U	90 mph			100 mph			110 mph			120 mph		
	α	30°	45°	60°	30°	45°	60°	30°	45°	60°	30°	45°	60°
$SF_{n=10}$	24	2.5	1.7	1.3	2.0	1.4	1.1	1.6	1.1	0.9	1.4	1.0	0.7
10	48	2.3	1.[illegible]	1.1	1.9	1.[illegible]	0.9	1.5	1.1	0.8	1.3	0.[illegible]	0.6
D_P, Z_P	72	2.2	1.5	1.0	1.8	1.2	0.8	1.4	1.0	0.7	1.2	0.8	0.6
1.0	96	2.0	1.4	0.9	1.7	1.1	0.7	1.4	0.9	0.6	1.1	0.8	0.5
3	120	1.9	1.3	0.8	1.6	1.0	0.7	1.3	0.9	0.5	1.1	0.7	0.5
$SF_{n=10}$	24	~~3.3~~	2.3	1.7	2.7	1.8	1.4	2.2	1.5	1.2	1.8	1.3	1.0
10	48	~~3.1~~	2.[illegible]	1.5	2.5	1.7	1.2	2.1	1.4	1.0	1.7	1.2	0.8
D_P, Z_P	72	2.9	1.9	1.3	2.3	1.6	1.1	1.9	1.3	0.9	1.6	1.1	0.8
1.0	96	2.7	1.8	1.2	2.2	1.5	1.0	1.8	1.2	0.8	1.5	1.0	0.7
4	120	2.6	1.7	1.1	2.1	1.4	0.9	1.7	1.1	0.7	1.4	1.0	0.6
$SF_{n=10}$	24	~~4.1~~	2.8	2.2	~~3.3~~	2.3	1.8	2.7	1.9	1.4	2.3	1.6	1.2
10	48	~~3.8~~	2.[illegible]	1.9	~~3.1~~	2.1	1.5	2.6	1.8	1.3	2.2	1.5	1.1
D_P, Z_P	72	~~3.6~~	2.4	1.7	2.9	2.0	1.4	2.4	1.6	1.1	2.0	1.4	0.9
1.0	96	~~3.4~~	2.3	1.5	2.8	1.8	1.2	2.3	1.5	1.0	1.9	1.3	0.8
5	120	~~3.2~~	2.1	1.4	2.6	1.7	1.1	2.2	1.4	0.9	1.8	1.2	0.8
$SF_{n=10}$	24	~~4.9~~	~~3.4~~	2.6	~~4.0~~	2.8	2.1	~~3.3~~	2.3	1.7	2.8	1.9	1.5
10	48	~~4.6~~	~~3.[illegible]~~	2.3	~~3.7~~	2.[illegible]	1.8	~~3.1~~	2.1	1.5	2.6	1.[illegible]	1.3
D_P, Z_P	72	~~4.3~~	2.9	2.0	~~3.5~~	2.4	1.6	2.9	2.0	1.3	2.4	1.6	1.1
1.0	96	~~4.1~~	2.7	1.8	~~3.3~~	2.2	1.5	2.7	1.8	1.2	2.3	1.5	1.0
6	120	~~3.9~~	2.5	1.6	~~3.1~~	2.1	1.3	2.6	1.7	1.1	2.2	1.4	0.9

For example, the safety factor for a 10-post array (n = 10) with a front height array at 120″ and thirty degrees, α = 30°, with a one-foot diameter pier, D_P = 1′, three feet deep, Z_P = 3′, the safety factor is 1.9, or just under SF = 2.0.

As of this writing (2/9/2022) I am leaning toward a recommendation of h = 48″and α = 60°. I have highlighted the safety factors for this pair of parameters for each of the four pier depths and each of the four windspeeds modeled. Of the sixteen resulting safety factors, the only result that achieves SF > 2.0 is Z_P = 6′, U = 90 mph. Of the sixteen results four [4] of these result in SF < 1.0, three of which are for Z_P = 3′ as indicated by red font on a red background. Based on these preliminary results prior to our planned soil testing, I recommend Z_P = 5′ to avoid the safety factor of less than unity at U = 120 mph, α = 60°, n = 10, D_P = 1′, Z_P = 4′.

Future work will include stress analysis for posts and rails as well as pull-out results for n = 12, 14, 16 and D_P = 1.5′ and 2.0′. I will also perform a stress analysis for at least three [3] common schedule-40 structural tubing standards.

Bradley Layton PhD PE